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LETTERS TO THE EDITOR

PREDICTION OF FREE–FREE MODES FROM SINGLE POINT SUPPORT GROUND VIBRATION TEST OF LAUNCH VEHICLES

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1. INTRODUCTION

Ground Vibration Tests (GVTs) play an important role in the development of launch vehicles by providing the experimental vibration information for the assembled structure. This information is important for validating many studies that predict the free vibration characteristics and whose results are used for the design of control system, apart from the assessment of overall structural dynamic response of the structure. Accuracy of the GVT depends on many factors such as correct translation of the design into the hardware form, accurate instrumentation and testing procedure. In view of the fact that the launch vehicle is a free structure during flight, its simulation as a free structure during the ground vibration test is also an important issue that needs considerable attention. In general, the ground tests [1, 2] on flight structures suffer from the basic handicap that these structures need to be supported in some way for the test, while providing as close to the free–free elastic boundary condition as possible [3]. These two conflicting requirements have led to the most universally acceptable configuration for the launch vehicle ground test in which the structure is supported on two points, corresponding to the nodal points of the fundamental mode of vibration. It is found that such an arrangement provides a reasonable estimate of the fundamental mode while the second and higher modes are only approximately known. Further, the two point support needs to be designed very carefully so that its influence on the higher mode is kept to a minimum, while adequately supporting a massive launch vehicle structure, and this leads to the standard test configuration in which a combination of stiff linear springs and soft rotational springs are used. Recently, the author has investigated the two point support mechanism for the ground test from the point of view of understanding the sensitivity of the vibration solution to the support parameters, e.g., location and its stiffness [4].

This study has demonstrated that the requirement of a very stiff linear and a soft rotational spring at nodal points makes the vibration results very sensitive to a shift in the support point. Further, a stiff linear spring at the first mode nodal points interferes significantly with the higher modes and provides erroneous estimates of their frequencies and mode shapes. In recent times, increased flexibility of the launch vehicle structure has made the higher modes also important from many considerations and there is a strong need to evolve a different ground vibration test procedure that is capable of providing reasonably accurate results for at least first three free–free vibration modes of the launch vehicle structure.

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The present study proposes to examine the concept of a single point clamping support for carrying out the ground vibration test on launch vehicles with a view to predicting the corresponding correct in-flight free–free modes.

The proposed procedure is of an empirical nature wherein the vibation results, obtained from tests conducted with structure clamped at its centre of gravity (i.e., c.g.), are processed using analytical predictions to provide the corresponding free–free results. It may be mentioned here that, although theoretically any point on the structure can be used as the clamping point for the test, the c.g. has the basic advantages of static mass balance which makes the support design easier and less massive. Further, during the flight the resultant of inertia forces acts through the centre of gravity, rendering it a pseudo reaction point and the deformation of two branches from the c.g. matches fairly closely with the actual free–free mode shape of the launch vehicle. Finally, it is found that for a large class of launch vehicles, the c.g. is generally close to either zero slope point (odd numbered modes) or zero displacement point (even numbered modes) and a suitable choice of support point stiffness can lead to a viable support configuration for the vibration test. In view of the above, the present study examines the applicability of single point support in the GVT of launch vehicles, for predicting accurately the free–free frequencies and mode shapes.

2. PROBLEM FORMULATION AND SOLUTION

Figure 1(a) shows the geometry of a general launch vehicle supported with a combination of a stiff linear and a soft rotational spring at its c.g.. Such a support can be realized by supporting the vehicle on two vertical beams kept very close to each other (see Figure 1(b)). In this case, the high axial stiffness of support beams provides a near zero displacement at the support point. However, the bending stiffness of the beam is much smaller in comparison and provides a marginal rotational restraint. In view of the fact that all roughly antisymmetric modes (even numbered) have large modal rotations around the c.g., it makes sense to use a softer rotational spring for beter prediction capability. In addition, all odd numbered modes have nearly zero modal slope around the c.g. and therefore, the presence of a small rotational restraint would not make much difference to the

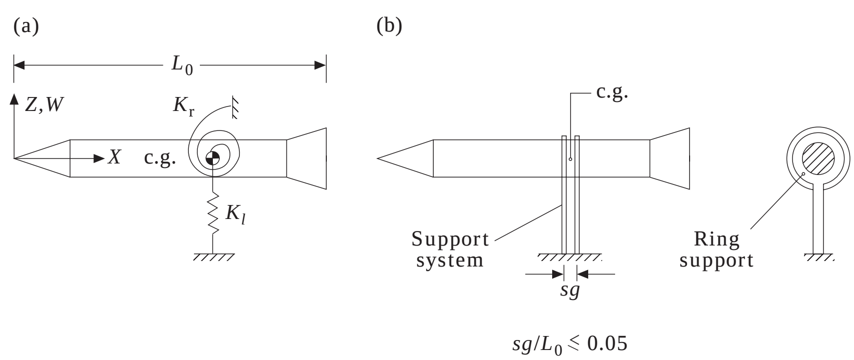

Figure 1. Geometry and coordinate system of a spcae vehicle supported as its c.g..

modal prediction accuracy of these modes. The harmonic transverse vibration of stepped slender beams can be adequately described, using elementary beam theory, by the following set of differential equations [4],

$$(\partial^4 w_i/\partial \bar{x}_i^4) + \gamma_i^4 w_i = 0, \tag{1}$$

where, i (=1–N) is the constant property beam segment identifier, w_i is the transverse deformation of the ith beam segment, $\bar{x}_i$ $(=x_i/L_0)$ is the span co-ordinate in the ith segment and γ_i^4 $(=[(\rho A)_i \omega^2 L_0^4/(EI)_i])$ is the dimensionless frequency parameter for the ith segment. A new frequency parameter for the complete launch vehicle can be defined as,

$$\lambda^4 = \rho A_0 \omega^2 L_0^4 / EI_0, \tag{2}$$

where, ρA_0, L_0 and EI_0 are reference values. General solution of equation (1) is,

$$w_i = A_i \cosh \gamma_i \bar{x}_i + B_i \sinh \gamma_i \bar{x}_i + C_i \cos \gamma_i \bar{x}_i + D_i \sin \gamma_i \bar{x}_i, \tag{3}$$

where A_i, B_i, C_i and D_i are arbitrary constants of integration. The following equations specify the continuity conditions between the beam segments, excepting the clamp point.

$$w_i(\overline{L}_i) - w_j(0) = w_i'(\overline{L}_i) - w_j'(0) = 0,$$
$$w_i''(\overline{L}_i) - w_j''(0) = w_i'''(\overline{L}_i) - w_j'''(0) = 0. \tag{4, 5}$$

At the clamp point, shear force and bending moment continuity is specified as,

$$w_i''(\overline{L}_i) - w_j''(0) - \overline{K}_r w_j'(0) = w_i'''(\overline{L}_i) - w_i'''(0) - \overline{K}_1 w_j(0) = 0, \tag{6}$$

where $\overline{K}_r(=K_r L_0/EI_0)$ and $\overline{K}_1(=K_1 L_0^3/EI_0)$ are rotational and linear spring constants at support point and index j is $i+1$. Finally, four free–free conditions on the two end points are,

$$w_1''(0) = w_1'''(0) = w_N''(\overline{L}_N) = W_N'''(\overline{L}_N) = 0. \tag{9}$$

The solution for zeros of the characteristic determinant of size $4N \times 4N$ gives values of λ in the present case. There is one branch of the launch vehicle each on either side of the support point, which vibrates at the single frequency parameter λ. This frequency is different from the actual free–free frequency and is used for making predictions of the corresponding free–free frequencies.

3. RESULTS FOR A UNIFORM UNITY BEAM STRUCTURE

Uniform beams are the most common modelling approximations to be employed while dealing with various issues of launch vehicle structural dynamics. In the present case a uniform unit beam (i.e., $EI_0 = 1{\cdot}0$, $\rho a_0 = 1{\cdot}0$ and $L_0 = 1{\cdot}0$) is assumed for the study of single point supported launch vehicles. The support configuration is such that its linear spring stiffness ratio $\overline{K}_1$ is kept constant at $1{\cdot}0 \times 10^8$ and the rotational spring stiffness ratio $\overline{K}_r$ is chosen steps of 0·1, 0·2, 0·5 and 1·0.

Table 1(a) presents the results for the frequency parameter λ for the above four cases, in addition to the free–free case, for the first six vibration modes and it is

TABLE 1

(a) Frequencies of the uniform unity vehicle

Mode	Case: $\bar{K}_l$ 0·0, $\bar{K}_r$ 0·0	$1{\cdot}0 \times 10^8$, 0·1	$1{\cdot}0 \times 10^8$, 0·2	$1{\cdot}0 \times 10^8$, 0·5	$1{\cdot}0 \times 10^8$, 1·0
1	4·730	3·750	3·750	3·750	3·750
2	7·853	7·847	7·841	7·823	7·791
3	10·99	9·388	9·388	9·388	9·388
4	14·14	14·13	14·13	14·12	14·10
5	17·28	15·71	15·71	15·71	15·71
6	20·42	20·42	20·41	20·41	20·40

(b) Frequencies of normalized generic space vehicle

Mode	Case: $\bar{K}_l$ 0·0, $\bar{K}_r$ 0·0	$1{\cdot}0 \times 10^8$, 1·0	$1{\cdot}0 \times 10^8$, 2·0	$1{\cdot}0 \times 10^8$, 5·0	$1{\cdot}0 \times 10^8$, 10·0
1	4·98	4·28	4·27	4·26	4·24
2	7·95	7·38	7·37	7·37	7·35
3	10·84	10·82	10·81	10·80	10·76
4	13·60	12·48	12·48	12·48	12·47
5	17·41	16·76	16·76	16·76	16·73
6	20·44	20·18	20·18	20·17	20·16

found that there is a clear distinction between the symmetric (i.e., odd numbered) modes and the antisymmetric (even numbered) modes. It is seen that the symmetric modes are not influenced by a change in the support rotational restraint while the antisymmetric modes are influenced marginally, only by the rotational restraint. This is because all symmetric modes have zero slope at the support point while all antisymmetric modes have zero deformation at the support point, rendering the supported mode shapes as functions of only one parameter (i.e., either $\bar{K}_l$ or $\bar{K}_r$). Table 1(a) shows that the symmetric frequencies of the supported configuration are much lower than the corresponding free–free values in comparison to the antisymmetric frequencies and this fact is brought out more clearly in Figures 2(a)–2(f), which present this comparison of mode shapes for the first six vibration modes. It is clearly seen that mode shape changes for the symmetric frequencies are much more significant than those for the antisymmetric frequencies. Interestingly, the region of modal mismatch for the supported beam in relation to the free–free beam is a function of the mode number and its width becomes smaller for higher modes, leading to a smaller percentage reduction in the frequencies for the higher symmetric modes and the difference between the free–free and the supported symmetric mode shape can be described adequately with the help of a simple sine function,

$$f_n(1) = (p_f - p_s) \sin \{(n+1)\pi 1/2L_0\}, \tag{12}$$

where, L_0 is the length of the beam, n is the mode number, p_f and p_s are the modal deformations at beam midpoint for the free–free and the supported case and l is the axial variable that varies only from 0–$\{2L_0/(n+1)\}$. A small part of the modal difference, outside the above range, is ignored by this function assuming it to be insignificant.

Figures 2(a), 2(c) and 2(e) show that both free–free and supported mode shapes have the same modal mass value and therefore, a reduction in the modal stiffness is directly responsible for the reduction in the frequency parameter (Table 1(a)). Hence, it is possible to arrive at the estimate of the loss in the modal stiffness for the supported beam, using the above functional form for $f_n(l)$ as [5],

$$\Delta K_n = \int EI(l) f_n(l) f_n(l)'''' \, \mathrm{d}l, \qquad (13)$$

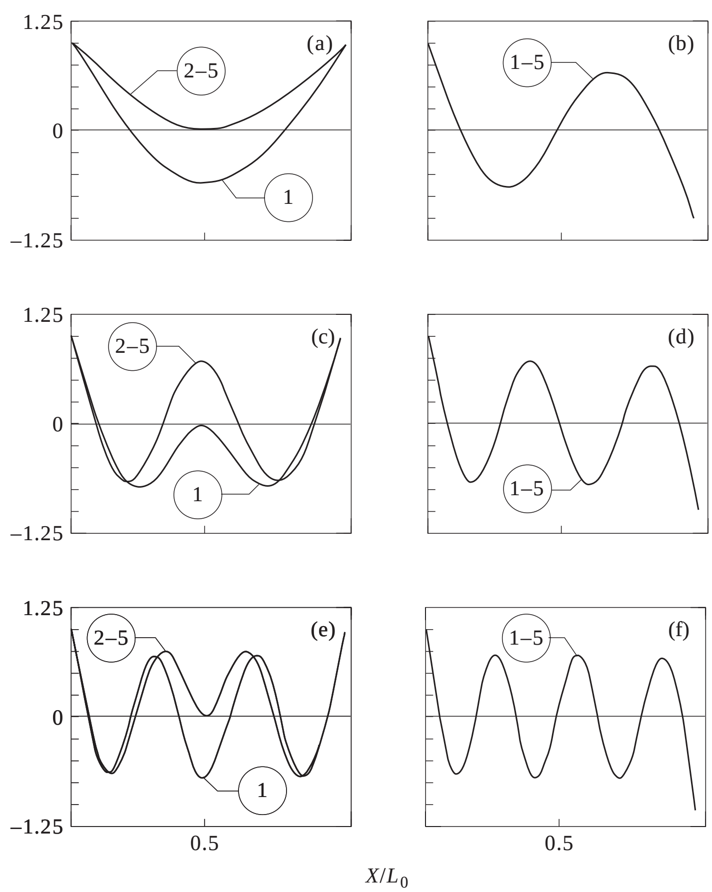

Figure 2. Normalized mode shapes of a uniform unity space vehicle for (1) free–free case, (2–5) supported case with $\bar{K}_1 = 1 \times 10^8$ and $\bar{K}_r = 0{\cdot}1$, $0{\cdot}2$, $0{\cdot}5$ and $1{\cdot}0$ respectively. (a)–(f); modes 1–6.

where ΔK_n is the stiffness loss in the nth mode and $EI(1)$ is the bending rigidity distribution. In the present case, the above parameter is obtained as

$$\Delta K_n = 4\pi^4(p_f - p_s)^2/(n+1). \tag{14}$$

The expression for the predicted free–free frequency can now be obtained from the supported frequency for the symmetric vibration modes as,

$$\lambda_{nf} = \lambda_{ns}[1 + \Delta K_n/(\lambda_{ns}^4 m_n)]^{1/4}, \tag{15}$$

where λ_{nf} is the nth predicted frequency parameter, λ_{ns} is the nth measured frequency parameter and m_n is the modal mass for the nth vibration mode. Figures 2(a), 2(c) and 2(e) show that the difference $(p_f - p_s)$ for the first three symmetric modes is nearly same with an average value of the order of 0·63, while the modal mass m_n is 0·25 for all the three symmetric modes. Therefore, the expression for λ_{nf}, given in equation (15), can be further simplified as,

$$\lambda_{nf} = \lambda_{ns}[1 + 623{\cdot}42/\{\lambda_{ns}^4(n+1)\}]^{1/4}, \qquad n = 1, 3, 5, \ldots . \tag{16}$$

The first three predicted symmetric vibration modes frequency parameter, λ, for the free–free conditions, have been calculated from the above relation as 4·75, 10·86 and 17·45 in comparison to the exact values which are 4·73, 10·99 and 17·28 (see Table 1(a)). This represents a maximum difference of about 1·2% which can be considered to be acceptable. The free–free mode shape can also be accurately predicted by superposing the difference function $f(1)$ onto the supported mode shape.

With regard to the anti-symmetric (i.e., even numbered) modes, it is seen from figures 2(b), (d) and (f) that the mode shapes are not altered at all even when the rotational parameter varies from 0–1·0. While the second mode frequency is marginally influenced by the variation in the rotational restraint, the fourth and sixth mode frequencies are practically unaffected. This indicates that a simple algebraic expression of the following form is capable of predicting the antisymmetric mode frequencies, for moderate values of the rotational restraint.

$$\lambda_{nf} = \lambda_{ns}[1 + 0{\cdot}122(\bar{K}_r/n^2)]^{1/4}, \qquad n = 2, 4, 6, \ldots . \tag{17}$$

The fit constant of 0·122 in the obove expression is obtained as an arithmetic average from the three anti-symmetric modes and to determine the adequacy of this expression, the second free–free mode frequency is calculated from the supported case with $\bar{K}_r = 0{\cdot}5$ and the value is obtained as 7·853 which is the exact result. Thus, the study for a uniform unity space vehicle has clearly brought out the fact that accurate predictions for the frequencies and mode shapes are possible for the first six free–free vibration modes, from the results obtained from a single point supported configuration.

4. RESULTS FOR A NORMALIZED SPACE VEHICLE STRUCTURE

Space vehicle structures are such that they are stiffer and heavier at the root and more flexible and ligher near the tip. Further, it is generally seen that while individual magnitudes may vary singificantly for different launch vehicles, the normalized variation of the stiffness and mass in the core vehicle is generally very

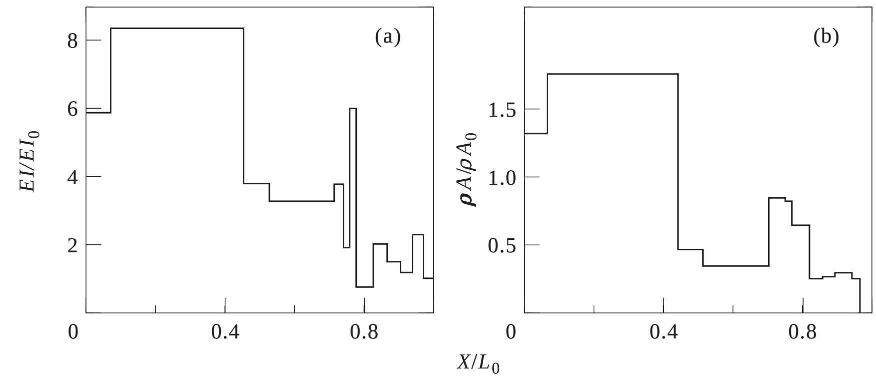

Figure 3. Normalized structural configuration in the form of (a) bending rigidity and (b) mass distributions for a generic space vehicle core structure.

similar to each other and it is possible to consider normalized space vehicle to be fairly generic. In the present case, the normalized launch vehicle structure described in reference [4] is used, whose summary is given in Figure 3. In this case also, the support configuration at the c.g. (at a normalized distance of 0·348 from the root) is taken as a combination of a stiff linear spring ($\bar{K}_1 = 1 \times 10^8$) and a set of soft rotational springs ($\bar{K}_r = 1{\cdot}0$, 2·0, 5·0 and 10·0), with the reference values for frequency normalization same as those for the uniform unity space vehicle. The ratio of the largest rotational spring constant and the maximum normalized bending rigidity is kept around unity, similar to the unity space vehicle. Table 1(b) presents the frequency parameter for the first six modes of the normalized generic launch vehicle and it is seen that the overall trends are similar to the ones observed for the uniform unity space vehicle. However, there are notable difference in the behaviour inasmuch as that all the six modes are influenced by the increase in the rotational restraint, because the c.g. is no longer on the zero slope point for any of the six modes. Similarly, the linear stiff support at the c.g. is not on either a node or a zero slope point, and thus there are no well defined trends visible in the results for the supported frequency in Table 1(b). A clearer picture emerges from the Figures 4(a)–4(f) which present the mode shapes of all the six vibration modes and it is found that, in general, the effect of support point on the frequency parameter λ, increases with increase in the normalized modal distance between the support point and the nearest nodal point (s_n).

However, the influence of s_n also varies with the mode number, while the generalized mass for the supported case (m_n) is different for different modes in such a way that it appears to be directly affecting the supported frequency. In the present case it is assumed that both s_n and m_n are influenced only by the location of the linear stiff spring and therefore s_n takes values as 0·149, 0·113, 0·007, 0·069, 0·034 and 0·017 and m_n takes values as 0·068, 0·025, 0·031, 0·045, 0·038 and 0·049, for the first six modes. These values are considered to be applicable also to the case when $\bar{K}_r$ is not zero. It is now possible to arrive at a simple expression for the free–free frequency parameter λ_{nf}, using the modal parameters s_n and m_n as follows.

$$\lambda_{nf} = \lambda_{ns} g(\bar{K}_r)[1 + 100\{s_n \quad m_n\}]^{1/4}, \qquad n = 1, 2, 3, \ldots . \tag{18}$$

The general expression is of the same form as that for the uniform unit space vehicle, with the difference that the same expression is valid for all the six modes. However, it is noted that the results for all the modes are now a function of the rotational restraint $\bar{K}_r$ and this dependence is denoted by the function $g(\bar{K}_r)$ as follows.

$$g(K_r) = 1 + 0{\cdot}009\{K_r/(nEI_m)\}, \qquad n = 1, 2, 3, \ldots, \tag{19}$$

where EI_m is the maximum value of the normalized $EI(x)$ distribution for the core vehicle. The fit constants 100 and 0·009 used in expressions (18) and (19) are arrived at by minimizing the least squares error across the different modes. Values of λ_{nf}, obtained for the case with $\bar{K}_r = 2{\cdot}0$ from above relations, are 5·09, 7·86, 10·89, 13·38, 17·31 and 20·63 while corresponding exact values are 4·98 7·95, 10·84, 13·60, 17·41 and 20·44. The maximum error in λ_{nf} prediction is found to be of the order of 2·2%, indicating acceptability of expressions (18) and (19). With regard to the prediction of the mode shape, it is seen from Figures 4(a–f) that it is not

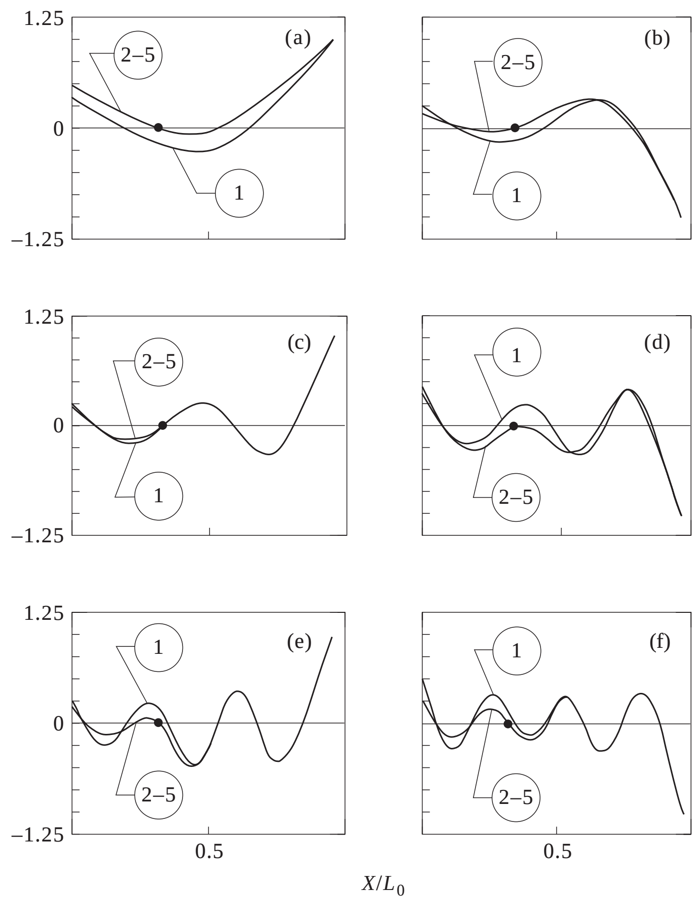

Figure 4. Normalized mode shapes of a normalized generic space vehicle for (1) free–free case and (2–5) supported case with $\bar{K}_1 = 1 \times 10^8$ and $\bar{K}_r = 1{\cdot}0$, 2·0, 5·0 and 10·0 respectively. (a)–(f); modes 1–6.

possible to define general functional forms. However, it can be seen that if the supported mode shape is shifted vertically, as a rigid body around the tip, by an amount (s_n/n, $n = 1, 2, 3, \ldots$), it is possible to arrive at a fairly good estimate for up to four free–free modes. The supported fifth and sixth modes require a more complex transformation to correctly predict the corresponding free–free mode shape.

5. CONCLUSIONS

In this study, the problem of accurate free–free mode prediction from a single point ground vibration test results for a space vehicle structure has been investigated. Elementary beam theory governing equations has been used to solve the problem of launch vehicle structure as a stepped beam. The vehicle is supported on its c.g. with the help of a combination of stiff linear spring and soft rotational spring, and exact beam function solution is extracted for two normalized vehicle cases. Firstly, a uniform unit beam is considered and the results show a clear distinction between the even and the odd numbered modes, for which simple and accurate algebraic expressions have been developed for predicting the free–free frequencies and mode shapes from a supported configuration. Next, a normalized generic space vehicle core structure was analyzed in the supported form and it has been found that there is no clear cut distinction between even and odd numbered modes. In this case also, simple and accurate expressions are derived for the corresponding free–free frequencies which depend on the modal mass, rotational restraint and the minimum modal separation between the free–free node location and the vehicle c.g.. Mode shapes for the first four modes can be adequately obtained by simple rigid body transformations. The study brings into focus, the potential for carrying out the ground tests on launch vehicles in more realistic and achievable support configurations and still being in a position to predict the first six free–free vibration frequencies and mode shapes fairly accurately.

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